For Grade 4

Some things change,
some remain the same

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Alwyn Olivier and Piet Human are the principal authors of the text in this booklet.
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1 NUMBER SENTENCES

1.1 State addition and subtraction facts

If something is not true, we say it is false.
For example, this sentence is false:

A bird has eight legs.

Sentences such as the following are called number sentences:

If you add 3 to 10 you get the same as when you add 4 to 9.

We can also write number sentences in symbols:

\[ 9 + 4 = 10 + 3 \]

1. Which of the following sentences are true, and which are false?
(a) Four apples and three apples, altogether, is seven apples.
(b) Six apples and three apples, altogether, is ten apples.
(c) Six apples and one apple, altogether, is seven apples.
(d) Five apples and two apples, altogether, is seven apples.

2. (a) How much is 5 + 5? 
(b) How much is 5 + 4?
(c) How much is 7 + 3?
(d) How much is 7 + 5?
(e) How much is 8 + 4?

3. Nathi says 5 cubes + 4 cubes is the same number of cubes as 7 cubes + 3 cubes.
He writes 5 + 4 = 7 + 3.
Is this true or false?

4. Mpho says 5 + 5 is the same number as 7 + 3.
She writes 5 + 5 = 7 + 3.
Is this true or false?
5. Decide which of the sentences below are false. If a sentence is false, make it true. Keep the left-hand side the same and change the right-hand side. Then write two more true number sentences with the same left-hand side.

Example: \(5 + 3 = 6 + 4\) is false.
But \(5 + 3 = 6 + 2\) is true.
\(5 + 3 = 2 + 6\) and \(5 + 3 = 1 + 7\) are also true number sentences.

(a) \(8 + 5 = 10 + 3\)  (b) \(8 + 6 = 7 + 7\)
(c) \(2 + 9 = 9 + 2\)  (d) \(80 + 70 = 100 + 20\)
(e) \(70 + 50 = 80 + 50\)  (f) \(19 − 5 = 20 − 4\)
(g) \(13 − 7 = 14 − 8\)  (h) \(13 − 7 = 15 − 9\)
(i) \(13 − 7 = 20 − 14\)  (j) \(20 + 8 = 10 + 18\)
(k) \(10 + 6 = 20 − 4\)  (l) \(30 + 17 = 40 + 7\)

The number sentence \(7 + 5 = 9 + 3\) can also be said in words, for example in any of the following ways:

*The sum of 7 and 5 is equal to the sum of 9 and 3.*
*If you add 5 to 7 you will get the same answer as when you add 3 to 9.*
*Seven plus five is equal to nine plus three.*

6. Write each of the following number sentences in words, in the three different ways shown above.

(a) \(7 + 9 = 10 + 6\)  (b) \(13 + 7 = 15 + 5\)
(c) \(19 − 5 = 20 − 6\)  (d) \(5 + 3 + 6 = 6 + 5 + 3\)
(e) \(4 + 4 + 4 + 4 + 4 = 6 + 6 + 6 + 6\)

7. Write each number sentence in symbols.

(a) The difference between 10 and 3 is equal to the sum of 5 and 2.
(b) If you subtract 8 from 13 you will get the same answer as when you subtract 10 from 15.
(c) Ten plus four is equal to eight plus six.
1.2 Equivalence

1. (a) Ben picked 50 mangoes. Later he picked 30 more mangoes. How many mangoes did he pick altogether?

(b) Sissy picked 30 mangoes in the morning. Later she picked 50 more mangoes. How many mangoes did she pick altogether?

When you add 30 to 50 you get the same answer as when you add 50 to 30.

We can write this number sentence in symbols:

\[ 50 + 30 = 30 + 50 \]

When you add two numbers, it does not matter which one you take first.

Adding 30 to 50 gives the same result as adding 50 to 30.

Lea, Ada and Piet have to calculate 20 + 30 + 50.

Lea plans to first calculate 20 + 30 and then add 50 to the answer.

Piet plans to first calculate 30 + 50 and then add the answer to 20.

Ada plans to first calculate 20 + 50 and then add 30 to the answer.

2. (a) Do you think Lea and Piet will get the same answer?

(b) Do you think Ada will also get the same answer?

All over the world, people sometimes use brackets to indicate which calculations they plan to do first.

Piet can write his plan like this: 20 + (30 + 50).

Ada can write her plan like this: (20 + 50) + 30.

3. How can Lea write her plan?
4. Below are four different plans to calculate $8 + 5 + 7$. The brackets indicate which calculations must be done first. Do each calculation in the way the plan states.

(a) $(8 + 5) + 7$  
(b) $8 + (5 + 7)$  
(c) $8 + (7 + 5)$  
(d) $(8 + 7) + 5$

5. Which other plans can be followed to calculate $8 + 5 + 7$?

Read this before you answer question 6:
In some cases in question 6, you may have to do the calculations to find out whether the two plans give the same result or not. The calculations given inside the brackets must be done first.

6. Which of the number sentences below are false? For each false number sentence, make a true number sentence by writing a different plan on the right-hand side.

(a) $(20 + 8) - 5 = 20 + (8 - 5)$
(b) $(20 - 8) - 5 = (20 - 5) - 8$
(c) $(20 - 8) - 5 = 20 - (8 - 5)$
(d) $(8 + 2) + (7 + 3) = (8 + 3) + (7 + 2)$
(e) $(8 + 7) + (2 + 3) = (8 + 3) + (7 + 2)$
(f) $(20 + 4) + (10 + 2) = (20 + 10) + (4 + 2)$

**When you have to add many numbers, it does not matter where you start.**

7. Which of the number sentences below are false?

(a) $63 + 26 = (60 + 20) + (3 + 6)$
(b) $8 + 5 = (8 + 2) + 3$
(c) $(20 + 7) - (10 + 4) = (20 - 10) + (7 - 4)$
(d) $(30 + 6) - (10 + 8) = (20 - 10) + (16 - 8)$
1.3 Describe patterns with number sentences

In each **row** in Diagrams A, B and C, the total number of cubes = the number of dark grey cubes + the number of light grey cubes. The rows go from left to right.

1. For the first row in Diagram A, you can write the number sentence $10 + 1 = 11$.
   For the second row you can write the number sentence $9 + 2 = 11$.
   Write number sentences for all the other rows.

   ![Diagram A](image)

Now look at the **columns** in Diagram A. The columns go from top to bottom.

2. For the first column in Diagram A, you can write the number sentence $10 + 0 = 10$.
   For the second column you can write the number sentence $9 + 1 = 10$.
   Write number sentences for all columns.

   ![Diagram B](image)

3. (a) Write number sentences for all the rows in Diagram B.
   (b) Write number sentences for all the columns in Diagram B.

4. How do Diagrams A and B differ?

5. (a) Write number sentences for all the rows in Diagram C.
   (b) Write number sentences for all the columns in Diagram C.
6. We can write the number sentence $10 + 10 = 20$ to describe the bottom row in Diagram D.

Write a number sentence for each of the other rows.

Diagram D

7. Write a number sentence for each row in Diagram E.

Diagram E

8. Write a number sentence for each row in Diagram F.

Diagram F

9. (a) There are more dark grey cubes in Diagram E than in Diagram D. How many more?

(b) There are more dark grey cubes in Diagram F than in Diagram E. How many more?

10. What is the same about Diagrams A and D, and what is different?
1.4 Solve and complete number sentences

The number sentence below is incomplete. One of the numbers is missing.

\[ 4 + 6 = 2 + ? \]

An incomplete number sentence is also called an open number sentence.

The number 8 will make the above number sentence true:

\[ 4 + 6 = 2 + 8 \]

The sentence \( 4 + 6 = 2 + 8 \) is called a closed number sentence.

Instead of a question mark, a little block \( \square \) or dots . . . or the word number may be used to write an open number sentence:

\[ 4 + 6 = 2 + \ldots \]  or  \[ 4 + 6 = 2 + a \text{ number} \]  or  \[ 4 + 6 = 2 + \square \]

1. In each case, find the number that will make the number sentence true.

(a) \( 7 + 3 = 5 + \ldots \)  
(b) \( 70 + 30 = 40 + \square \)  
(c) \( 700 + 300 = 800 + \ldots \)  
(d) \( 80 + 50 = 80 + 20 + \square \)  
(e) \( 7 + 9 = 10 + \ldots \)  
(f) \( 75 + \ldots = 100 \)  
(g) \( \ldots + 500 = 1000 \)  
(h) \( 120 + \ldots = 150 + 50 \)  
(i) \( \ldots + 750 = 1000 \)  
(j) \( 487 + \ldots = 500 \)

2. (a) Find two different numbers that will make this number sentence true:

\[ 8 + a \text{ number} = 10 + a \text{ different number} \]

(b) Find two other numbers that will also make the above number sentence true.

(c) Find another two numbers that will make the above number sentence true.
3. Complete the number sentences:
   (a) $3 + 7 = \ldots$
   (b) $30 + 70 = \ldots$
   (c) $300 + 700 = \ldots$
   (d) $3 + 6 = \ldots$
   (e) $30 + 60 = \ldots$
   (f) $300 + 600 = \ldots$
   (g) $2 + 6 = \ldots$
   (h) $20 + 60 = \ldots$
   (i) $200 + 600 = \ldots$
   (j) $4 + 6 = \ldots$
   (k) $40 + 60 = \ldots$
   (l) $400 + 600 = \ldots$
   (m) $3 + 5 = \ldots$
   (n) $30 + 50 = \ldots$
   (o) $300 + 500 = \ldots$
   (p) $3 + 4 = \ldots$
   (q) $30 + 40 = \ldots$
   (r) $300 + 400 = \ldots$
   (s) $9 + 4 = \ldots$
   (t) $90 + 40 = \ldots$
   (u) $80 + 40 = \ldots$
   (v) $8 + 5 = \ldots$
   (w) $80 + 50 = \ldots$
   (x) $70 + 40 = \ldots$

4. Complete the number sentences:
   (a) $10 - 3 = \ldots$
   (b) $100 - 30 = \ldots$
   (c) $1000 - 300 = \ldots$
   (d) $9 - 3 = \ldots$
   (e) $90 - 30 = \ldots$
   (f) $900 - 30 = \ldots$
   (g) $8 - 3 = \ldots$
   (h) $80 - 30 = \ldots$
   (i) $800 - 300 = \ldots$
   (j) $7 - 3 = \ldots$
   (k) $70 - 30 = \ldots$
   (l) $700 - 300 = \ldots$
   (m) $7 - 4 = \ldots$
   (n) $70 - 40 = \ldots$
   (o) $700 - 400 = \ldots$
   (p) $8 - 4 = \ldots$
   (q) $80 - 40 = \ldots$
   (r) $800 - 400 = \ldots$
   (s) $9 - 4 = \ldots$
   (t) $90 - 40 = \ldots$
   (u) $900 - 400 = \ldots$
   (v) $10 - 4 = \ldots$
   (w) $100 - 40 = \ldots$
   (x) $1000 - 400 = \ldots$

5. Complete the number sentences:
   (a) $9 + 5 = \ldots$
   (b) $90 + 50 = \ldots$
   (c) $190 + 50 = \ldots$
   (d) $14 - 5 = \ldots$
   (e) $140 - 50 = \ldots$
   (f) $240 - 50 = \ldots$
   (g) $13 - 5 = \ldots$
   (h) $130 - 50 = \ldots$
   (i) $230 - 50 = \ldots$
   (j) $430 - 50 = \ldots$
   (k) $430 - 60 = \ldots$
   (l) $430 - 70 = \ldots$
   (m) $8 + 7 = \ldots$
   (n) $80 + 70 = \ldots$
   (o) $60 + 70 = \ldots$
   (p) $15 - 8 = \ldots$
   (q) $150 - 80 = \ldots$
   (r) $750 - 80 = \ldots$
   (s) $13 - 8 = \ldots$
   (t) $130 - 80 = \ldots$
   (u) $430 - 80 = \ldots$
   (v) $12 - 7 = \ldots$
   (w) $120 - 70 = \ldots$
   (x) $130 - 70 = \ldots$
2 NUMERIC PATTERNS

2.1 Seeing patterns

The more patterns you can see in mathematics, the better you are at mathematics!

How good are you at seeing patterns?

1. In each case, say which item does not fit with the others. Explain why you say so.
   (a) 5, 10, 30, 45, 56, 25, 55, 20, 35
   (b) 12, 36, 10, 48, 23, 32, 40, 66, 24, 98
   (c) abc, bca, cab, bac, aca, acb, cba
   (d) A B C D E F G

2. Here are Lindi’s answers for question 1. Do you agree with her? Or did you see different patterns?
   (a) The numbers are all different. But what is the same is that they are all multiples of 5; except 56, which is not.
   (b) The numbers are all different. But what is the same is that they are all even numbers; except 23, which is an odd number.
   (c) The “words” are all different. But what is the same is that they all have the letters a, b and c, in any order; except aca which does not have a b.
   (d) The figures are all different. But what is the same is that they all have 4 sides; except Figure E, which has 5 sides.

Different, but the same!
To see a pattern, we look for what is the same in ALL the different things.
2.2 Making patterns

A **sequence** is a special number pattern in a row of numbers, where every number in the row is calculated with some **rule**. The same rule is repeated for every next number.

Here are two examples of sequences that you already know:

- When the teacher asks Sally to “count in fives”, she counts like this by adding 5 each time:
  5, 10, 15, 20, 25, ...

- For the instruction “start at 1 and count on in fives”, Sally counts like this:
  1, 6, 11, 16, 21, ...

We use the following words and notation to write and to talk about sequences. For example, in the case of the first sequence above:

```
<table>
<thead>
<tr>
<th>2nd number</th>
<th>4th number</th>
</tr>
</thead>
<tbody>
<tr>
<td>5, 10, 15, 20, ...</td>
<td></td>
</tr>
</tbody>
</table>
```

Three dots means it goes on forever.

We can also write the sequence in a table and indicate the **position number**, like this:

<table>
<thead>
<tr>
<th>Position number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequence</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>25</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
When learning about sequences, you will learn how to *recognise* patterns, *describe* patterns, and *continue* patterns.

You will learn how to answer questions such as these. You can try to answer them now, if you like.

- If Sally continues counting in the same way, what are the next two numbers she will count in each sequence?
- What will be the 100th number Sally will count in each sequence?
- If Sally continues and continues, will the number 325 be in her sequences or not? How do you know?

1. For each of the instructions in (a) to (f) below:
   - write down the sequence according to the instruction.
   - describe in your own words what is different and what is the same for all the numbers in the sequence.
   - write down the 100th number in the sequence.

   (a) Count in fives.
   (b) Start at 1 and count on in fives.
   (c) Start at 2 and count on in fives.
   (d) Start at 3 and count on in fives.
   (e) Start at 4 and count on in fives.
   (f) Start at 5 and count on in fives.

2. What is different, and what is the same for *all* the sequences in question 1?

3. Write down your own instruction for a sequence, and then write down the sequence.
2.3 Describing patterns

1. When a son was born, his father was 30 years old. How old is the father when the son is 10 years old?

Over the years, the son’s mother wrote down the father’s and the son’s ages in a table, like this:

<table>
<thead>
<tr>
<th>Age of son (years)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>12</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age of father (years)</td>
<td>30</td>
<td>31</td>
<td>32</td>
<td>33</td>
<td>34</td>
<td>35</td>
<td>__</td>
<td>__</td>
</tr>
</tbody>
</table>

2. Describe any patterns that you can see in the table. (What is different and what remains the same?)

3. Complete the table. This means:
   
   (a) Work out how old the father will be when the son is 12.
   
   (b) Work out how old the father will be when the son is 20.

We can describe the father and son’s ages in different ways:

- In **words**:
  
  *The father is always 30 years older than the son.*

- As a **calculation plan (rule)**:

  *Father’s age = Son’s age + 30*

  Although their ages change all the time, the calculation plan remains the same for all ages:

  - **Son’s age**
    
    1 \(\rightarrow\) 2 \(\rightarrow\) 3 \(\rightarrow\) 4 \(\rightarrow\) \(\ldots\) 10 \(\rightarrow\) 11
  
  - **Father’s age**
    
    31 \(\rightarrow\) 32 \(\rightarrow\) 33 \(\rightarrow\) 34 \(\rightarrow\) \(\ldots\) 40 \(\rightarrow\) 41

- With a **flow diagram**, such as the one on the next page.
4. Find the three missing output numbers; this means the father’s age when the son was 12, 15 and 20.

5. How do you calculate the input number for the output number 60? This is the same as finding the missing number in the open number sentence \( \square + 30 = 60 \). Calculate it.

6. Find the son’s age when the father is 37, 40, 43, 46 and 50.

7. This flow diagram shows the ages of another father and son.

(a) What is the calculation plan (operator) connecting the input and output numbers?

(b) Find the missing input and output numbers.
2.4 Recognising and describing patterns

1. Theo’s mother often sends him to buy candles for their home. The candles cost R4 each.

(a) Complete this table to calculate the cost of different numbers of candles.

(b) Describe your method.

<table>
<thead>
<tr>
<th>No. of candles</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (rands)</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>

2. This is Theo’s method:

<table>
<thead>
<tr>
<th>No. of candles</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (rands)</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Is Theo’s method correct?

(b) Describe his method in words.

(c) Use Theo’s method to calculate the cost of 6, 7 and 20 candles.

3. This is Nadia’s method:

<table>
<thead>
<tr>
<th>No. of candles</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (rands)</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(a) Is Nadia’s method correct?

(b) Describe her method in words.

(c) Use Nadia’s method to calculate the cost of 6, 7 and 20 candles.

We can describe the cost of the candles in different ways:

- In **words**:  
  *The candles cost R4 each.*
- As a **calculation plan (rule)**:  
  *Cost of candles = Number of candles × 4*
- With this **flow diagram**:

<table>
<thead>
<tr>
<th>Input numbers</th>
<th>Output numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of candles</td>
<td>Cost (R)</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>?</td>
</tr>
<tr>
<td>15</td>
<td>?</td>
</tr>
<tr>
<td>?</td>
<td>68</td>
</tr>
</tbody>
</table>

5. Calculate the missing numbers in the flow diagram.

6. This flow diagram shows the cost of candles at another shop.

<table>
<thead>
<tr>
<th>Input numbers</th>
<th>Output numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of candles</td>
<td>Cost (R)</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td>3</td>
<td>?</td>
</tr>
<tr>
<td>15</td>
<td>?</td>
</tr>
<tr>
<td>?</td>
<td>98</td>
</tr>
</tbody>
</table>

(a) What is the calculation plan connecting the input and output numbers?

(b) Find the missing input and output numbers.
### 2.5 Tables or multiples

Here is part of the multiplication table.

<table>
<thead>
<tr>
<th>×</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>8</td>
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<td>5</td>
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<td>6</td>
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<td>18</td>
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</tr>
<tr>
<td>7</td>
<td>7</td>
<td>14</td>
<td>21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Complete the table.

2. Which methods did you use to complete the table? Discuss.

3. Discuss what patterns you see in the table, and how that helps you to “remember” the tables.

The multiplication table consists of sequences of **multiples**. For example:

2, 4, 6, 8, 10, 12, ... are the **multiples of 2** (or even numbers).

3, 6, 9, 12, 15, ... are the **multiples of 3**.

These sequences are all different, but they are the same in two ways:

- In all the “tables” or sequences of multiples we add the same number (the multiple) to get the next number in the sequence. For example:

  \[
  \begin{array}{cccccccc}
  2 & 4 & 6 & 8 & 10 & 12 & \ldots \\
  +2 & +2 & +2 & +2 & +2 & +2 & \\
  \end{array}
  \]

  \[
  \begin{array}{cccccccc}
  3 & 6 & 9 & 12 & 15 & 18 & \ldots \\
  +3 & +3 & +3 & +3 & +3 & +3 & \\
  \end{array}
  \]
We can, for example, describe the sequence 3, 6, 9, 12, ... by saying that we add 3 to each number to get the next number. So, we can also use it as a calculation plan.

- The sequences of multiples all have the same kind of calculation plan (rule), namely multiplication only:

<table>
<thead>
<tr>
<th>Position no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple no.</td>
<td>×2</td>
<td>×2</td>
<td>×2</td>
<td>×2</td>
<td>×2</td>
<td>×2</td>
<td>...</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Position no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple no.</td>
<td>×3</td>
<td>×3</td>
<td>×3</td>
<td>×3</td>
<td>×3</td>
<td>×3</td>
<td>...</td>
</tr>
</tbody>
</table>

So, we can describe the sequence 3, 6, 9, 12, ... with a calculation plan:

\[
\text{Multiple number} = \text{Position number} \times 3
\]

So: multiple 100 = 100 × 3 = 300

4. (a) Calculate the next five numbers and the 100th number in each sequence. Describe your methods.

Sequence A: 2, 4, 6, 8, 10, 12, 14, 16, ...

Sequence B: 3, 6, 9, 12, 15, 18, 21, ...

Sequence C: 5, 10, 15, 20, 25, 30, 35, ...

Sequence D: 7, 14, 21, 28, 35, 42, 49, ...

Sequence E: 9, 18, 27, 36, 45, 54, 63, ...

Sequence F: 10, 20, 30, 40, 50, 60, 70, ...

(b) What is the same and what is different in each and in all of these sequences?
3 GEOMETRIC PATTERNS

3.1 Our geometric art heritage

In this unit we study geometric patterns, such as the decorations on the walls of the homes of the Ndebele people in Mpumalanga.

The Ndebele like to decorate the outside of their homes with colourful designs.

The Ndebele also make and wear beautiful bead necklaces and bracelets.

They love geometric patterns! Do you?

A geometric pattern is a repeated decorative design.

In this unit we will not focus on the types of figures (such as triangles, rectangles, and so on) but on the number of figures or beads in such repeating patterns.
3.2 Clever counting

1. (a) How many beads are there in this string? Explain your method.

(b) Look at these methods that other learners used. Who is correct? Which method do you think is the best?

Simon points with his finger and counts the beads one by one: “One, two, three ... 28, 29.”

Amir says: “There are five white beads in each group and three gray beads. So I count 5, 8, 13, 16, 21, 24, 30, 33.”

Mia says: “There are five white beads in each group and three gray beads. So I count the white beads first: 5, 10, 15, 20. Then I continue to count the gray beads: 23, 26, 29, 32.”

Thea says: “I group the white beads and the gray beads together like this and then count in eights: 8, 16, 24, 32.”

2. Use clever counting to easily find the number of beads in each of the four bracelets below. Explain how you do it. Compare your method with some of your classmates’ methods.

Simon counted the beads one by one. The other learners counted bigger units like 3 and 5 and 8. They used clever counting!
3.3 Do not count – calculate!

When we count many objects it can take a very long time and we may make mistakes!

A better way is to first write down your thinking as a calculation plan. Write down what you are going to do.

Then you can calculate the answer instead of counting. For example:

Amir: Number of beads = 5 + 3 + 5 + 3 + 5 + 3 + 5 + 3
Mia: Number of beads = 5 + 5 + 5 + 5 + 3 + 3 + 3 + 3 = 4 × 5 + 4 × 3
Thea: Number of beads = 8 + 8 + 8 + 8 = 4 × 8

For each bracelet on the next page, write down your calculation plan to find out:
(a) how many white beads there are
(b) how many gray beads there are
(c) how many beads there are altogether.

Note: Do not calculate now. The method (plan) is the important thing! You can calculate later or even use a calculator once you have written down your plans.

A plan describes your method. You can write a plan in words, as a calculation plan or as a flow diagram.

4 × 5 + 4 × 3 and 4 × 8 and 5 + 3 etc. are calculation plans. A calculation plan says what calculations we are going to do before we actually do them.
1. Lindiwe is making this growing pattern of pictures with squares:

![Picture 1]  
Picture 1

![Picture 2]  
Picture 2

![Picture 3]  
Picture 3

![Picture 4]  
Picture 4

![Picture 5]  
Picture 5

Lindiwe plans to continue the pattern.

(a) Describe Picture 6 and Picture 7 in words. Now draw Picture 6 and Picture 7. How many squares are there in Picture 6 and how many in Picture 7?

(b) Describe Picture 60 and Picture 70 in words. Do not draw them! Imagine them; “see” them in your head! Calculate the number of squares in Picture 60 and in Picture 70.

3.4 Growing patterns
2. Simphiwe is making these growing patterns of pictures with squares.

**Pattern A**

![Pattern A Pictures]

**Pattern B**

![Pattern B Pictures]

**Pattern C**

![Pattern C Pictures]

**Pattern D**

![Pattern D Pictures]

Answer questions (a) and (b) for each of Patterns A, B, C and D.

(a) Describe Picture 6 and Picture 7 in words.

Then draw Picture 6 and Picture 7.

How many squares are there in Picture 6 and how many in Picture 7?

(b) Describe Picture 60 and Picture 70 in words. Do not draw them! Imagine them; “see” them in your head!

Calculate the number of squares in Picture 60 and in Picture 70.
3.5 From pictures to tables

Elspeth is making this growing pattern of pictures with light gray and dark gray triangles.

![Figures 1 to 4](image)

Figure 1 Figure 2 Figure 3 Figure 4

1. Complete this table and describe your methods.

<table>
<thead>
<tr>
<th>Figure number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of light gray triangles</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of dark gray triangles</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total no. of triangles</td>
<td>4</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Describe the number patterns you see in the table.

3. Describe Figure 30 in words.
   (a) How many light gray triangles are there in Figure 30?
   (b) How many dark gray triangles are there in Figure 30?
   (c) How many triangles are there in Figure 30?

4. If Elspeth makes a figure with 31 light gray triangles, how many dark gray triangles does the figure have?

5. If Elspeth makes a figure with 31 dark gray triangles, how many light gray triangles does the figure have?
3.6 Writing our plans as flow diagrams

You have already seen this growing pattern in Section 7.4. We will now look at it again, but in a different way. How will you calculate the number of squares in Picture 6, Picture 60 and Picture 87? What is your plan?

![Picture 1 to Picture 5]

Mia writes her plan as a flow diagram:

A flow diagram describes a plan (method) using input → rule → output.

This flow diagram shows that you must first multiply any input number by 2 and then add 4 to get the output number.

\[
\begin{align*}
3 \times 2 & \rightarrow 6; \ 6 + 4 & \rightarrow 10 \\
4 \times 2 & \rightarrow 8; \ 8 + 4 & \rightarrow 12
\end{align*}
\]

1. First check if Mia’s flow diagram (plan) is correct for the input and output numbers that we already know. Discuss with your classmates how you can check her plan.
2. If Mia’s plan is correct, use her plan to calculate the missing output numbers.

3. Mia says, with the flow diagram it is easy to calculate the number of squares in Picture 60 or in Picture 87 or in any Picture number. Do you agree?

4. If you or your classmates have a different plan than Mia, write it as a flow diagram.

5. Write your plans for each of these patterns as flow diagrams, and calculate the number of squares in Picture 6, Picture 60 and Picture 87. Then compare your methods with some classmates’ methods.

(a) 

Picture 1  Picture 2  Picture 3  Picture 4  Picture 5

(b) 

Picture 1  Picture 2  Picture 3  Picture 4  Picture 5

6. Make your own growing geometric pattern with squares.

(a) Draw the first four pictures.

(b) Calculate the number of squares in Picture 5, Picture 6, Picture 50 and Picture 60.

(c) Now let some classmates solve your problem in (b) and check if they are correct.
4 Numeric Patterns

4.1 Patterns in times tables

<table>
<thead>
<tr>
<th>×</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
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<td>10</td>
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<td>2</td>
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<td>8</td>
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<td>20</td>
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<td>16</td>
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<td>28</td>
<td>32</td>
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<td>24</td>
<td>30</td>
<td>36</td>
<td>42</td>
<td>48</td>
<td>54</td>
<td>60</td>
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<tr>
<td>7</td>
<td>7</td>
<td>14</td>
<td>21</td>
<td>28</td>
<td>35</td>
<td>42</td>
<td>49</td>
<td>56</td>
<td>63</td>
<td>70</td>
</tr>
</tbody>
</table>

You know that in all horizontal and vertical sequences in the multiplication table, the same number is added to get the next number in the sequence. Here is an example:

\[ 4 \quad 8 \quad 12 \quad 16 \quad 20 \]

\[ +4 \quad +4 \quad +4 \quad +4 \]

We say there is a **constant difference** between consecutive numbers. For example:

\[ 8 - 4 = 4; \quad 12 - 8 = 4; \quad 16 - 12 = 4; \ldots \]

We will now look at other sequences in the multiplication table, and see what patterns they have.
1. Find this sequence in the multiplication table on page 262.
1, 4, 9, 16, 25, 36, 49, ...
(a) How will you describe this sequence? What horizontal calculation plan (going from one number to the next) is used to make each next number in the sequence?
(b) Continue the sequence for another five numbers.
(c) Can you find a vertical calculation plan describing how the numbers are made?

<table>
<thead>
<tr>
<th>Position no.:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequence no.:</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>36</td>
<td>...</td>
</tr>
</tbody>
</table>

(d) Calculate the 20th and the 100th number in the sequence. Did you use a horizontal plan or did you use a vertical plan? Which one is easier?

2. Find this sequence in the multiplication table:
2, 6, 12, 20, 30, 42, 56, ...
(a) How will you describe this sequence? What horizontal calculation plan (going from one number to the next) is used to make each next number in the sequence?
(b) Continue the sequence for another five numbers.
(c) Can you find a vertical calculation plan describing how the numbers are made?

<table>
<thead>
<tr>
<th>Position no.:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequence no.:</td>
<td>2</td>
<td>6</td>
<td>12</td>
<td>20</td>
<td>30</td>
<td>42</td>
<td>...</td>
</tr>
</tbody>
</table>
(d) Calculate the 20th and the 100th number in the sequence. Did you use a horizontal plan or did you use a vertical plan? Which one is easier?

3. Find this sequence in the multiplication table:
3, 8, 15, 24, 35, 48, 63, ...

(a) How will you describe this sequence? What horizontal calculation plan is used to make each next number in the sequence?

(b) Continue the sequence for another five numbers.

(c) Can you find a vertical calculation plan describing how the numbers are made?

<table>
<thead>
<tr>
<th>Position no.:</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequence no.:</td>
<td>3</td>
<td>8</td>
<td>15</td>
<td>24</td>
<td>35</td>
<td>48</td>
<td>...</td>
</tr>
</tbody>
</table>

(d) Calculate the 20th and the 100th number in this sequence. Did you use a horizontal plan or did you use a vertical plan? Which one is easier?

4.2 Tables, rules and flow diagrams

You know that for the sequence 4, 8, 12, 16, 20, 24, ... we have horizontal and vertical patterns with which we can continue the pattern, for example:

\[
\begin{array}{cccccccc}
1 & 2 & 3 & 4 & 5 & 6 & \cdots & 100 \\
\times4 & \times4 & \times4 & \times4 & \times4 & \times4 & \times4 \\
4 & 8 & 12 & 16 & 20 & \cdots \\
+4 & +4 & +4 & +4 & +4 & \end{array}
\]

But can you find the 100th number in 5, 9, 13, 17, 21, ...?
1. Below are two tables and two flow diagrams. Which flow diagram is equivalent to which table (gives the same output numbers for the same input numbers)? Calculate all the missing numbers.

<table>
<thead>
<tr>
<th>Input numbers</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output numbers</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Input numbers</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output numbers</td>
<td>5</td>
<td>9</td>
<td>13</td>
<td>17</td>
<td>21</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Flow Diagram](image)

![Flow Diagram](image)
2. For each of the Sequences A to E below:
   (a) Write a flow diagram for the sequence.
   (b) Continue the sequence for another five numbers.
   (c) Calculate the 100th number in the sequence.
   (d) How are the sequences the same and different?
       How are the flow diagrams the same and different?

   Sequence A:  4, 8, 12, 16, 20, 24, 28, ...
   Sequence B:  5, 9, 13, 17, 21, 25, 29, ...
   Sequence C:  6, 10, 14, 18, 22, 26, 30, ...
   Sequence D:  7, 11, 15, 19, 23, 27, 31, ...
   Sequence E:  8, 12, 16, 20, 24, 28, 32, ...

3. For each of the Sequences A to F below:
   (a) Write a calculation plan (rule) for the sequence.
   (b) Continue the sequence for another five numbers.
   (c) Calculate the 100th number in the sequence.
   (d) How are the sequences the same and different?
       How are the rules the same and different?

   Sequence A:  5, 10, 15, 20, 25, 30, ...
   Sequence B:  6, 11, 16, 21, 26, 31, ...
   Sequence C:  7, 12, 17, 22, 27, 32, ...
   Sequence D:  8, 13, 18, 23, 28, 33, ...
   Sequence E:  12, 17, 22, 27, 32, 37, ...
   Sequence F:  4, 9, 14, 19, 24, 29, ...
4.3 Computer sequences

To make Tables P, Q, R and S, a computer used Rules A, B, C and D given below.

Rules (calculation plans)

A \[ Output \text{ number} = 3 \times Input \text{ number} + 4 \]

B \[ Output \text{ number} = 4 \times Input \text{ number} + 3 \]

C \[ Output \text{ number} = 2 \times Input \text{ number} + 5 \]

D \[ Output \text{ number} = 5 \times Input \text{ number} + 2 \]

Tables

<table>
<thead>
<tr>
<th>P</th>
<th>Input numbers</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Output numbers</td>
<td>7</td>
<td>12</td>
<td>17</td>
<td>22</td>
<td>27</td>
<td>57</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Q</th>
<th>Input numbers</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>100</th>
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<tr>
<td></td>
<td>Output numbers</td>
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<td>9</td>
<td>11</td>
<td>13</td>
<td>15</td>
<td>57</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>R</th>
<th>Input numbers</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tbody>
<tr>
<td></td>
<td>Output numbers</td>
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<td>11</td>
<td>15</td>
<td>19</td>
<td>23</td>
<td>55</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>S</th>
<th>Input numbers</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Output numbers</td>
<td>7</td>
<td>10</td>
<td>13</td>
<td>16</td>
<td>19</td>
<td>58</td>
<td></td>
</tr>
</tbody>
</table>

1. Which rule did the computer use for which table? Match the rules and the tables. Describe how you know and how you can be sure that you are right.

2. Complete the tables for the missing input and output numbers.

3. Draw a flow diagram for each of the four rules. Insert the operators, input and output numbers.
5

NUMBER SENTENCES

5.1 Learn to use number sentences

Sometimes it is easy to see which calculations you have to do to find information, for example:

*Ben has R120 and he pays R50 for food. How much money does he have left?*

Sometimes it is more difficult to see which calculations you must do, for example:

*Bettina spent R60 on food and then she had R80 left. How much money did she have before she bought the food?*

In a case like this, it may help to write a number sentence to understand what you must do. We can write:

*The money Bettina had − 60 = 80*

A number sentence like this will help you to see that in this case you can calculate 80 + 60 to find out how much money Bettina had.

80 + 60 = 140 so she had R140.

To check, you can put your answer into the number sentence: 140 − 60 = 80, so R140 is the right answer.

1. Answer the questions that you find easy. Skip the other questions.

(a) Gwede has 60 goats. He buys more goats and now he has 75 goats. How many goats did he buy?
(b) Zweli has 75 goats. He buys another 60 goats. How many goats does he have now?

(c) Lerato has 75 goats. This is 60 goats more than Willem has. How many goats does Willem have?

2. Find the missing number in each of these number sentences.
(a) $75 - \ldots = 60$  
(b) $\ldots + 60 = 75$  
(c) $60 + \ldots = 75$
(d) $60 + 75 = \ldots$  
(e) $75 - 60 = \ldots$  
(f) $75 + 60 = \ldots$
(g) $\ldots - 60 = 75$  
(h) $\ldots - 75 = 60$

3. Go back to the parts of question 1 that you skipped because you found them difficult. For each one, find a number sentence in question 2 that can help you. Try to answer all the parts of question 1 now.

4. Answer the questions that you find easy. Skip the other questions.

(a) Ishmael has 75 goats and Simon has 60 goats. How many more goats does Ishmael have than Simon?

(b) Pieter had 75 goats. He sold 60 goats. How many goats does he have left?

(c) Lettie had 75 goats. Some goats were stolen and now she has only 60 goats left. How many goats were stolen?

(d) Moses buys 60 goats and now he has 75 goats. How many goats did he have before he bought more goats?

(e) Johan sold 60 goats and now he has 75 goats left. How many goats did he have before he sold some of his goats?

(f) Mpho has 60 goats and her sister Nellie has 75 goats. How many goats do the two sisters have altogether?

5. Go back to the parts of question 4 that you skipped because you found them difficult. For each one, find a number sentence in question 2 that can help you. Try to answer all the parts of question 4 now.
5.2 Use number sentences

For some of the questions below you will know immediately which calculations to do. For other questions you may need to write a number sentence first.

1. There are 78 learners on two buses together. There are 42 learners on the one bus. How many learners are on the other bus?

2. There are 2 378 learners in two schools together. There are 1 426 learners in the one school. How many learners are there in the other school?

3. After 5 478 chickens were killed in a storm, Nomvula had 3 243 chickens left. How many chickens did she have before the storm?

4. Jamie paid R384 for a pair of shoes and Vusi paid R423 for his shoes. How much more did Vusi pay than Jamie?

5. Peter bought 3 chairs for his house, all at the same price. He also bought a refrigerator for R2 780. Peter paid R3 677 in total. How much did each of the chairs cost?

6. Gertie walks 184 m to school and back home every day, and Simon walks 124 m to school and back home. How much further than Simon does Gertie walk to school and back, in five days?
7. John is 143 cm tall and Janet is 157 cm tall. How much taller than John is Janet?

8. Ma Minah bought 3 chickens for R44 each. She also bought a bag of potatoes. Ma Minah paid R174 in total. How much did the potatoes cost?

5.3 Try a number and improve

1. (a) Is the number sentence below true if you put 6 in both places where a number is missing?

$$5 \times a \text{ number} + 8 = 3 \times \text{the same number} + 22$$

(b) Is the number sentence true if you put 10 in both places?

(c) Is it true if you put 7 in both places?

2. (a) Which of these numbers do you think will make the number sentence below true?

$$6 \times a \text{ number} + 10 = 10 \times \text{the same number} − 26$$

(b) Put the number you think will work in the two places and calculate to see if it works. If it does not work, try another number.

(c) Continue until you find a number that makes the number sentence true.
3. In each case, try different numbers until you find the number that makes the number sentence true:

(a) $20 \times \text{the number} + 40 = 30 \times \text{the same number} + 10$

(b) $23 \times \text{the number} - 60 = 15 \times \text{the same number} + 20$

(c) $6 \times \text{the number} + 5 = 10 \times \text{the same number} - 55$

(d) $6 \times \text{the number} + 5 = 10 \times \text{the same number} - 47$

(e) $6 \times \text{the number} + 5 = 10 \times \text{the same number} - 39$

(f) $400 - 10 \times \text{the number} = 4 \times \text{the same number} + 50$

(g) $8 \times \text{the number} + 14 = 10 \times \text{the same number} - 20$

(h) $37 \times \text{the number} + 15 = 15 \times \text{the same number} + 37$
6.1 Investigate and extend patterns

1. (a) Describe in words how this growing pattern of triangles is made.

Triangle 1  Triangle 2  Triangle 3  Triangle 4

(b) Describe Triangle 6 and Triangle 7 in words. How many dots are there in Triangle 6 and how many in Triangle 7?

(c) Describe Triangle 60 and Triangle 70 in words. Calculate the number of dots in Triangle 60 and in Triangle 70.

(d) Complete this table. Describe and discuss your methods.
Describe and discuss what patterns you see in the table.

<table>
<thead>
<tr>
<th>Triangle number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of dots</td>
<td>3</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. On the next page you can see growing patterns of squares, pentagons and hexagons. Investigate each pattern by answering these questions.

(a) Describe in words how the growing pattern is made.

(b) Describe Figure 6 (that is Square 6, Pentagon 6 and Hexagon 6) and Figure 7 in words. How many dots are there in Figure 6 and how many in Figure 7?
(c) Describe Figure 60 and Figure 70 in words. Calculate the number of dots in Figure 60 and in Figure 70.

(d) Complete this table. Describe and discuss your methods. Describe and discuss what patterns you see in the table.

<table>
<thead>
<tr>
<th>Figure number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of dots in square</td>
<td>4</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of dots in pentagon</td>
<td>5</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of dots in hexagon</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. How are the patterns of triangles, squares, pentagons and hexagons the same, and how are they different?
6.2 Investigate and extend more patterns

1. (a) Describe in words how this growing pattern of triangles is made.

Triangle 1  Triangle 2  Triangle 3  Triangle 4

(b) Describe Triangle 6 and Triangle 7 in words. How many dots are there in Triangle 6 and how many in Triangle 7?

(c) Complete this table. Describe and discuss your methods. Describe and discuss what patterns you see in the table.

<table>
<thead>
<tr>
<th>Triangle no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of dots</td>
<td>3</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. (a) Describe in words how this growing pattern of squares is made.

Square 1  Square 2  Square 3  Square 4

(b) Complete this table. Describe and discuss your methods. Describe and discuss what patterns you see in the table.

<table>
<thead>
<tr>
<th>Square no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of dots</td>
<td>4</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>30</td>
</tr>
</tbody>
</table>

3. How are these patterns different from those in Section 10.1?
6.3 From tables to flow diagrams

![Figure 1](image1) ![Figure 2](image2) ![Figure 3](image3) ![Figure 4](image4)

Figure 1  Figure 2  Figure 3  Figure 4

1. Complete this table. Describe and discuss your methods. Describe and discuss what patterns you see in the table.

<table>
<thead>
<tr>
<th>Figure number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of light gray triangles</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of dark gray triangles</td>
<td>3</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total number of triangles</td>
<td>4</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Make a flow diagram to show how the number of light gray triangles can be calculated. Use the Figure numbers as the input numbers and the number of light gray triangles as the output numbers. Complete all missing parts.

<table>
<thead>
<tr>
<th>Figure no.</th>
<th>No. of light gray triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

Calculation plan

Input numbers  Output numbers

3. Repeat question 2, but this time make a flow diagram to show how the number of dark gray triangles can be calculated.

4. Repeat question 2 again but show how the total number of triangles can be calculated.